AIAA 2005-1359
Assessment of Incompressible Formulations for Numerical Solutions of Unsteady Turbulent Flows over Bluff Bodies
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43rd AIAA AeroSpace Sciences Meeting and Exhibit
January 10-13 2005 / Reno, NV
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American Institute of Aeronautics and Astronautics
Assessment of Incompressible Formulations for Numerical Solutions of Unsteady Turbulent Flows over Bluff Bodies

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Two algorithms commonly used for solving low-speed flow fields are evaluated using an unsteady turbulent flow formulation. The first algorithm is the method of artificial compressibility which solves the incompressible Navier-Stokes equations. The second is a preconditioned system for solving the compressible Navier-Stokes equations. Both algorithms have been implemented into GASP Version 4, which is the flow solver used in this investigation. Unsteady numerical simulations of 2-D flow over square cylinders are performed with comparisons made to experimental data. Formulations include both a single- cylinder and a three-cylinder configuration. Two turbulence models are also used in the computations, namely the Spalart-Allmaras model and the Wilcox k-$\omega$ (1998) model. The following output data was used for comparison: aerodynamic forces, mean pressure coefficient, Strouhal number, velocity magnitude and turbulence intensity. The main results can be summarized as follows; first, the predictions are more sensitive to the turbulence model choice than to the choice of algorithm. Second, using the Wilcox k-$\omega$ model, the preconditioned Roe algorithm produced more consistent results with experiment. Lastly, the Spalart-Allmaras model overall produced better results with both algorithms than the Wilcox k-$\omega$ model.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
</tr>
<tr>
<td>$C_P$</td>
<td>pressure coefficient</td>
</tr>
<tr>
<td>$D$</td>
<td>cylinder side length</td>
</tr>
<tr>
<td>$h$</td>
<td>height of test section</td>
</tr>
<tr>
<td>$n$</td>
<td>frequency</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number</td>
</tr>
<tr>
<td>$TKE$</td>
<td>Turbulence kinetic energy</td>
</tr>
<tr>
<td>$U$</td>
<td>$x$-direction local velocity</td>
</tr>
<tr>
<td>$V$</td>
<td>free-stream velocity</td>
</tr>
</tbody>
</table>

I. Introduction

Computational fluid dynamics (CFD) technologies have become a vital part of design and analysis in both research and commercial industries. CFD offers the ability to make a very detailed and thorough study of flow-fields and is capable of solving a broad range of flow problems from incompressible, low-speed, low-Reynolds-number to compressible, high-speed, high-Reynolds-number flows. Low-speed flows pose problems when solving the

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compressible Navier-Stokes equations. First, the length scales that are to be resolved by the numerical grid tend to be dissimilar. Second, the equations are highly non-linear, which leads to thin internal layers with time varying position. Finally, the incompressibility constraint must be satisfied at all times, which implies coupling at each time step. Historically, incompressible low-Reynolds number flows were solved by pressure-based algorithms. This approach decouples the set of non-linear, governing equations in which a pressure Poisson equation is formed from the momentum and continuity equations. PISO and SIMPLE are some examples of algorithms that employ this approach. In more recent years, density-based algorithms that employ time-marching techniques have grown in popularity. These algorithms solve for all the unknowns as a coupled system.

This research is concerned with the numerical simulation of turbulent, unsteady, low-speed flow using density-based algorithms implemented into the GASP Version 4 flow solver. Two such algorithms are discussed here. The first is the method of artificial compressibility which was introduced by Chorin. This method is commonly used in industry and research for solving complex, incompressible low-speed flows. The modifications to the standard Roe flux scheme used here are based on the preconditioning matrix of Weiss and Smith. The preconditioned flux differs from the standard flux in the acoustic corrections which are appended to the average flux. The preconditioned Roe flux can handle both low-speed and high-speed flows. It can be applied to steady and unsteady flows, as well as chemically reacting (variable density) flows. Specific details of both the artificial compressibility method and the preconditioned Roe method implemented in GASP can be found in Ref [7].

When performing CFD simulations in which the Reynolds Averaged Navier-Stokes (RANS) equations are solved, the choice of turbulence model is critical because it contains some of the largest uncertainties in modeling. There are a variety of turbulence models used in modern CFD analysis to help better simulate the actual physics of the problem. An ideal model should introduce the minimum amount of complexity while capturing the essence of the relevant physics. Since an ideal model has yet to be found, two different “levels” of turbulence models are analyzed in this research. These include the one-equation Spalart-Allamaras model and the two-equation Wilcox k-ω model. These models were chosen because each consists of a different level of complexity, as well as computational efficiency. These models are run with the two different low-speed algorithms, and the results are compared with each other and experimental data in order to determine how accurately the flow is described by each model.

In order to provide a critical assessment of the two low-speed formulations and how they interact with the turbulence models considered, we have selected the problem of turbulent flow over a 2-D cylinder with a square cross-section. This problem is of interest for both practical and scientific reasons. From a practical viewpoint, the problem has application to a wide variety of cases including flow over buildings, cars and bridges as but a few examples. It is of interest from a scientific viewpoint, since it involves large-scale unsteadiness and separated flows. We consider both an isolated square cylinder and a vertical row of three square cylinders to widen the practical scope of our studies and to include mutual interference effects.

II. Computational Details

A. GASP Flow Solver

The GASP Version 4 flow solver is a time-dependent, three-dimensional RANS solver. It solves the integral form of the governing equations using an upwind-based, finite-volume formulation. Steady state solutions are marched in time using local time stepping, while time-accurate flows are solved using a dual-time stepping procedure. The dual-time stepping method incorporates a second temporal derivative for converging the root of the time-accurate discretization. This makes the dual-time step method appropriate for use with the artificial compressibility and preconditioned Roe algorithms, since both require time-stepping with non-physical time steps, which can destroy temporal accuracy when implemented in traditional time-dependent formulations.

An advantage of using the dual-time stepping method is that it has no stability restrictions concerning the time step. So, large physical time steps can be chosen so long as the steady-state solution in pseudo-time is converged,
which does have a stability restriction on the pseudo-time step. For the time-accurate simulations presented here, the Newton sub-iteration scheme was used, which is a subset of the dual-time stepping procedure. For Newton sub-iterations, the pseudo-time step is set to infinity, and the physical time step is then adjusted to insure stability.

B. Single-Cylinder Configuration

The first of two configurations to be discussed is uniform approach flow over a single square cylinder. The grid used was created using Gridgen. The grid is two-dimensional, which models an infinitely long square-cylinder placed in a confined flow. The model is 0.0254m in length on a side. The structured grid, with approximately 60,000 cells, was clustered around the surface of the cylinder in order to capture the boundary layer effects. The far-field boundary extended 40+ diameters away from the square cylinder. Figure 1 shows the grid close to the square cylinder. The boundary condition applied around the surface of the cylinder was a no-slip, adiabatic wall. Around the perimeter of the far-field boundary, a relaxed pressure inflow/outflow boundary condition was applied. The sides of the grid were first-order extrapolation. Some of the flow parameters are as follows:

- Free-stream Velocity: 7.65 m/s
- Inlet Mach Number: 0.022
- Density: 1.226 kg/m³
- Temperature: 297 K
- Turbulence Intensity: 0.05
- Reynolds Number: 1.3x10⁴

The Reynolds number was selected in order to match experimental data. The single square cylinder results for $C_p$, $St$, $C_L$ and $C_D$ are compared to experimental data from Lee and Norberg, which can be found in Table 1. The flow solver was run as 2nd order accurate with implicit, dual-time stepping. The time step was selected such that 120 cycles were performed per vortex shedding period (time step = $2 \times 10^{-4}$). A total of four simulations were run for this case. The artificial compressibility and preconditioned Roe algorithms were used for both the Spalart-Allmaras and the Wilcox $k-\omega$ turbulence models. The algorithms were run using 3rd order MUSCL extrapolation with no limiting.

C. Three-Cylinder Configuration

The second configuration is based on a turbulence generating experiment performed at Virginia Tech. The experiment was conducted in a low-speed wind tunnel and consisted of a turbulence generator upstream of the test section. A detailed view of the grid around the middle cylinder can be found in Figure 2, and a schematic of the tunnel and three-cylinder configuration can be found in Figure 3. The generator had three square cylinders, 0.0254 m in height (each bar) and extended the horizontal length of the generator. The cylinders were spaced vertically in the test section. The data from the experiments include the velocity magnitude and the turbulence intensity downstream of the cylinders. The data from the experiment is

<table>
<thead>
<tr>
<th>Author</th>
<th>Reynolds Number</th>
<th>Mean $C_L$</th>
<th>Mean $C_D$</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norberg</td>
<td>1.3x10⁴</td>
<td>0.00</td>
<td>2.15</td>
<td>0.132</td>
</tr>
<tr>
<td>Lee</td>
<td>1.7x10⁴</td>
<td>0.02</td>
<td>2.04</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Table 1. Experimental results for single square cylinder.
compared to CFD simulations presented here.

The grid was created using Gridgen10, and it mimicked the experimental wind tunnel setup. The tunnel duct was 1.7158m long and varied in height throughout. The inflow was 0.254m tall and expands to 0.381m at the location of the square cylinders. Downstream of the cylinders, the tunnel contracts to 0.3048m tall at the test section. The grid had approximately 90,000 cells. The structured grid was clustered near the walls in order to capture the boundary layer development on the cylinder surfaces and the top and bottom surfaces of the tunnel.

The boundary conditions applied to the three-cylinder configuration were similar to those of the single-cylinder configuration. A no-slip, adiabatic wall was applied to the surfaces of the cylinders, as well as the top and bottom tunnel walls. The inflow was set using subsonic flow conditions with relaxed pressure, and the outflow was set using a back pressure. Some of the key flow parameters are as follows:

- Free-stream Velocity: 20 m/s
- Inlet Mach Number: 0.064
- Density: 1.226 kg/m³
- Temperature: 297 K
- Turbulence Intensity: 0.05
- Reynolds Number: 2.7x10⁴

This case was run initially using the 1998 Wilcox k-ω turbulence model and artificial compressibility. The flux algorithm was then altered to preconditioned Roe and run for the same amount of time. The turbulence model was not changed, because an analysis of the incompressible algorithms was the focus of this case. The flow solver was run as 2⁰ order accurate with implicit dual-time stepping. The time step was selected such that 380 cycles were performed per vortex shedding period (time step = 2 x 10⁻⁵). The algorithms were run using 3⁰ order MUSCL extrapolation¹³ with no limiting.

III. Results

A. Single Square Cylinder

Results are now presented showing comparisons with experimental data, as well as comparisons between the different flow algorithms and turbulence models. This section will discuss results consisting of: aerodynamic force coefficients, flow field visualization, mean pressure coefficient, Strouhal number and an inner cycle convergence test.

1. Aerodynamic Force Coefficients: $C_D$ (X-Force) and $C_L$ (Y-Force)

The mean $C_D$ and mean $C_L$ data can be found in Table 2. The mean drag coefficients for the Spalart-Allmaras model agree more closely with experimental data than the drag coefficients for the Wilcox k-ω model. The result for the Spalart-Allmaras model is slightly lower (3%) than the experimental data, while the Wilcox k-ω model result is on average 15% higher than experimental data. The flux algorithm predictions did not vary significantly when the same turbulence model was used. The $C_D$ and $C_L$ data was averaged over several time periods; but, as can be seen from Figures 4 and 5, the forces did not behave in exact sinusoidal patterns. Therefore, the time to complete a period varied as did the height of each peak. Thus when $C_L$ was averaged, it was not exactly zero, as would be expected theoretically.
The magnitude of the oscillatory forces and the frequencies show a variation between the results from different turbulence models. To better visualize this, Figures 4 and 5 show sample increments in the time history of both the lift and drag coefficients with different flux schemes and turbulence models. The $C_D$ plot shows that the results do not follow a simple pattern and that the data is inconsistent between both turbulence models and algorithms (because of the chaotic and inconsistent behavior of $C_D$, only the mean value was compared to experiment, see Table 2). Figure 4 shows that a simple time-dependent $C_D$ pattern is hard to achieve using different algorithms and turbulence models. But, based on the information given in Table 2, the average $C_D$ predicted for all four formulations was off by no more than 15%.

The $C_L$ results plotted in Figure 5 are almost sinusoidal. When the $C_L$ data is averaged, it should be approximately zero since the cylinder is not angled with respect to the flow. As was shown and explained earlier, the averaged data is close but not exactly zero. The information that can be extrapolated from the $C_L$ results plot is the frequencies of vortex shedding and differences between the magnitude ranges of the four formulations. The two cases run with the artificial compressibility algorithm have the same range in magnitude but the frequencies are different. In the plot, the artificial compressibility algorithm with the Wilcox k-ω model shows slightly more than one sine wave, whereas the artificial compressibility algorithm with the Spalart-Allmaras model has slightly more than two waves. The two cases with preconditioned Roe do not match each other in frequency or magnitude range. However, one of those cases, the preconditioned Roe algorithm and the Spalart-Allmaras model, has peak magnitudes close to the artificial compressibility algorithm with the same turbulence model (Spalart-Allmaras). The frequencies are similar as well; both cases have just over two waves. It can be deduced that the cases run with the Spalart-Allmaras model produce waves with similar frequencies and that along with the artificial compressibility with Wilcox k-ω model, produce similar ranges in $C_L$ values. Only the mean $C_L$ data was compared with the experimental results.

Looking at both plots, the case using the preconditioned Roe algorithm with the Wilcox k-ω model overpredicts the forces when compared to the other cases. The results from the other three approaches stay approximately within the same force values.

### Table 2. Mean $C_D$ and $C_L$ for single cylinder configuration.

<table>
<thead>
<tr>
<th>Turbulence Model and Algorithm</th>
<th>Mean $C_D$</th>
<th>%Error of $C_D$</th>
<th>Mean $C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spalart-Allmaras</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preconditioned Roe</td>
<td>1.98</td>
<td>2.94</td>
<td>-0.0403</td>
</tr>
<tr>
<td>Artificial Compressibility</td>
<td>1.97</td>
<td>3.43</td>
<td>-0.0178</td>
</tr>
<tr>
<td><strong>Wilcox k-ω</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preconditioned Roe</td>
<td>2.32</td>
<td>13.7</td>
<td>-0.029</td>
</tr>
<tr>
<td>Artificial Compressibility</td>
<td>2.40</td>
<td>17.6</td>
<td>-0.026</td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lee</td>
<td>2.04</td>
<td>-</td>
<td>0.020</td>
</tr>
</tbody>
</table>

2. Flow Field Visualization

An instantaneous snapshot of vortex shedding produced by all the formulations can be seen in Figure 6 (a)-(d). Each plot shows the vorticity distribution at a peak in the $C_L$ data (force history data can be seen in Figure 5). All four figures show similar results, a large high pressure region in front of the cylinder, separation immediately off the
leading edge and periodic shedding in the wake. Visually, there appears to be a slight difference between the results with the Spalart-Allmaras model and the Wilcox k-ω model. The Wilcox k-ω simulations produced stronger intensity vortices than the Spalart-Allmaras model. Also, lateral vortices can be found in the separation region of the Wilcox k-ω model contours. The high pressure region in front of the square cylinder oscillates slightly in phase with the oscillations produced by the vertical shedding behind the cylinder. The flow is low speed, so the behavior downstream is expected to affect the upstream flow. Overall, the basic physics of the flow was captured in all four cases.

3. Strouhal number

The Strouhal number was calculated using equation 1.

\[ St = \frac{nD}{V} \]  
(Eq. 1)

where \( n \) is the frequency of the vortex shedding, \( D \) is the side length of the cylinder and \( V \) is the free-stream velocity. Table 3 lists the results from the calculations and the experimental data. The Spalart-Allmaras results are in good agreement with the Norberg experiment, but are larger than the experiment performed by Lee. This is most

Figure 6. Vorticity distribution at maximum force for single cylinder configuration.
likely to due the difference in Re between Lee and the CFD (see Table 1). The St results for the Wilcox k-ω model cases vary greatly from the experimental data (20+/% smaller). There is also a higher uncertainty in the St data for k-ω since the St value is very sensitive to the range of data used to compute it. The amount of uncertainty with the Wilcox k-ω turbulence model could be a result of the non-repetitive behavior of the forces. The St predictions are consistent with the force results (specifically, frequency results) found in Figures 4 and 5. The Wilcox k-ω model greatly under predicts the St, while the Spalart-Allmaras model results are only one percent less than the experimental St.

<table>
<thead>
<tr>
<th>Model</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spalart-Allmaras</td>
<td></td>
</tr>
<tr>
<td>Preconditioned Roe</td>
<td>0.130</td>
</tr>
<tr>
<td>Artificial Compressibility</td>
<td>0.131</td>
</tr>
<tr>
<td>Wilcox k-ω</td>
<td></td>
</tr>
<tr>
<td>Preconditioned Roe</td>
<td>0.118</td>
</tr>
<tr>
<td>Artificial Compressibility</td>
<td>0.104</td>
</tr>
<tr>
<td>Experimental</td>
<td></td>
</tr>
<tr>
<td>Norberg</td>
<td>0.132</td>
</tr>
<tr>
<td>Lee</td>
<td>0.122</td>
</tr>
</tbody>
</table>

4. Mean Pressure Coefficient on Surface

In order to focus on the effect the flux algorithms have on the mean pressure coefficient, $C_P$, the turbulence models were not compared. Instead, only the two low-speed algorithms are focused on here. Since the Spalart-Allmaras model produced the most repetitive data that was consistent with experimental force data, this turbulence model was used while varying the flow algorithm. The mean $C_P$ was measured from the center of the windward face, moving clockwise around the square cylinder. Figure 7 shows that the CFD data followed the same trend as the experimental data. The magnitudes of the data differed from experiment in two locations, the upper back corner and the lower back corner of the cylinder. The results from both algorithms followed the same pattern as the experiment. The preconditioned Roe results were slightly lower than the artificial compressibility result on the top and bottom sides of the cylinder. The discrepancies in data could be a result of using a small time period over which the CFD data was averaged or the difference between 2-D modeling and 3-D experiment.

5. Inner Cycle Convergence Test

Several cases were run while varying the number of inner cycles in the dual-time stepping algorithm. Again, only Spalart-Allmaras was used for a turbulence model, while the flux algorithms were compared. Artificial compressibility showed that 10 inner cycles produced different results than 20 and 30 inner cycles. Since 20 and 30 inner cycles do not differ in $C_D$ (and only vary slightly in $C_L$), it can be determined that 20 inner cycles is sufficient when using artificial compressibility. The preconditioned Roe algorithm produced the same pattern of results. In conclusion, both flux algorithms had similar convergence behavior.

<table>
<thead>
<tr>
<th>Inner Cycles</th>
<th>$C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artificial Compressibility</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.9659</td>
</tr>
<tr>
<td>20</td>
<td>1.9668</td>
</tr>
<tr>
<td>30</td>
<td>1.9668</td>
</tr>
<tr>
<td>Roe with Preconditioning</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.9491</td>
</tr>
<tr>
<td>20</td>
<td>1.9837</td>
</tr>
<tr>
<td>30</td>
<td>1.9837</td>
</tr>
</tbody>
</table>

B. Three–Cylinder Configuration

Results are now presented for flow over the three-cylinder configuration. All simulations performed on the three-cylinder configuration used the 1998 Wilcox k-ω turbulence model. The low-speed methods were varied between artificial compressibility and preconditioned Roe. A series of time-averaged resolved velocities and turbulence
intensities are computed and compared with experimental data. The aerodynamic forces and the Strouhal number are also calculated, and the results from the two flux algorithms are compared with each other. Experimental data was not available for the forces or the St for comparison, but only for the velocity and turbulence intensity profiles.

1. Aerodynamic Forces: Drag and Lift

The lift and drag forces on the grid were integrated over each cylinder and the results can be found in Figures 8 and 9. There is large difference in the behavior of drag predicted for the two algorithms. The preconditioned Roe algorithm result has a larger range in magnitude as well as a lower frequency than the artificial compressibility algorithm. The mean drag value differed by approximately 3%. The artificial compressibility algorithm produced 0.162 N of drag, and the preconditioned Roe algorithm produced 0.157 N of drag. The lift for both computations is sinusoidal and averages to zero, but the lift computed by the preconditioned Roe flux has a lower frequency (as with the drag force) than the lift computed by the artificial compressibility algorithm. The lift ranges from -0.06 to 0.06 N using the artificial compressibility algorithm and -0.09 to 0.09 N using the preconditioned Roe algorithm. The artificial compressibility algorithm predicted a more sinusoidal behavior, but both algorithms were able to predict an average force within 3% of each other.

2. Flow Field Visualization

A snapshot of the unsteady behavior can be seen in Figure 10 (a)-(b). Both flux algorithms produced similar visualization pictures. All three cylinders shed vortices periodically. The outer two cylinders are opposite in rotation,
but their frequencies are approximately the same. They both shed towards the center or towards the wall simultaneously. The middle cylinder cyclically sheds vortices with a slightly higher frequency than the outer cylinders. The vortices tend to break up sooner with artificial compressibility than with preconditioned Roe. Similar to the single cylinder case, there is a high pressure region in front of each cylinder, and the flow separates at the leading edge. The three cylinder case appears to have a tighter shedding region than the single cylinder case, and this could be due to the confinement of the three cylinders between two walls. The vorticity produced by both algorithms looks very similar to each other, and both capture the basic physics of the flow.

3. Average Velocity Magnitude

The predicted mean velocity variation across the duct at the measurement station is compared to experimental data in Figure 11. The velocity magnitude in the test section from the case using the artificial compressibility algorithm was approximately 23 m/s, which is slightly higher than predicted with the preconditioned Roe algorithm, which was 20 m/s and the experimental data at 18 m/s. The preconditioned Roe algorithm result was within 10% of the experimental data. The experimental data was taken only over part of the duct, whereas the CFD data is given from top wall to bottom wall. The artificial compressibility result agrees with the preconditioned Roe result better in the middle of the test section. Both algorithms over predict the average velocity downstream of the cylinders. The artificial compressibility algorithm result shows peaks in the velocity closer to the walls of the tunnel; which is not predicted by the preconditioned Roe solution.

4. Turbulence Data

The predicted turbulence intensities were calculated using equation 2. Figure 12 gives the data sets.

\[ ti = \frac{\sqrt{0.8 \times TKE}}{U} \]  

(Eq. 2)

The turbulence intensities predicted by the case with artificial compressibility were similar to the preconditioned Roe results and the experimental data. However, there is still approximately a 10% - 15% difference between the results for the two cases. The artificial compressibility algorithm predicted a higher peak value, 12% intensity, but averaged close to 11%. Preconditioned Roe results correlate well with the experimental data, which was steady along 10% intensity. Closer to the walls the turbulence intensity drops. The artificial compressibility algorithm again over predicts the experimental data and is inconsistent near the tunnel walls.

5. Strouhal Number

The Strouhal number was calculated using Eq. 1 for each cylinder. The St number results can be found in Table 5. The outside cylinders had the same value, while the middle cylinder produced a higher St for both algorithms. The algorithms predicted values were in good agreement with each other. These results agree with the trends predicted in Figure 9. The lift plot shows that the artificial compressibility algorithm result has a higher
frequency, which agrees with Table 5. These values were slightly higher than the St predicted for the single cylinder configuration (the three cylinder configuration was run with a higher Re). Moreover, the algorithms predict similar results for St.

<table>
<thead>
<tr>
<th>Model</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preconditioned Roe</td>
<td></td>
</tr>
<tr>
<td>Middle Cylinder</td>
<td>0.154</td>
</tr>
<tr>
<td>Outer Cylinders</td>
<td>0.142</td>
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<tr>
<td>Artificial Compressibility</td>
<td></td>
</tr>
<tr>
<td>Middle Cylinder</td>
<td>0.156</td>
</tr>
<tr>
<td>Outer Cylinder</td>
<td>0.144</td>
</tr>
</tbody>
</table>

6. Case Studies: Compressibility Parameter and Inner Cycles

In an effort to achieve a reasonable agreement between the results from the artificial compressibility and the preconditioned Roe algorithms, several studies were performed, such as an inner cycle convergence study and a study of the compressibility parameter, β. As a rule of thumb a good value for β is 5*V^2 where V is the free-stream velocity; but β was increased to 10*V^2 in an attempt for more accurate results. The results were consistent with a β=5, but the solution converged at a faster rate with β=10, so it was used.

A case study of inner cycles was also performed. Inner cycles of 5, 10 and 20 were used. It was determined that the 5 cycles per time step was sufficient for convergence for both flux schemes, so it was used. The single cylinder case used 20 cycles per step, but it was also run with a larger time step.

IV. Conclusions

CFD results for unsteady, turbulent, low-speed flow around square cylinders with varying formulations is presented. Flow predictions from two different low-speed algorithms as well as two turbulence models were compared. The flow algorithms studied consisted of the artificial compressibility method applied to the incompressible RANS and a preconditioned Roe scheme for the compressible RANS. Both algorithms are shown to be viable options for solving low-speed, unsteady flows using a dual-time stepping procedure. The turbulence models used in this study were the one-equation Spalart-Allmaras model and the two-equation k-ω model by Wilcox. Solutions were shown to be more sensitive to the turbulence model selection over the choice of flow algorithm.

The first configuration studied consisted of flow over a single square cylinder. Drag force and Strouhal numbers were compared to experimental data, and these comparisons showed that better correlations resulted from using the Spalart-Allmaras model. Varying the flow algorithm had much less impact on the solution than the turbulence model. Lift force history from the turbulence models consisted of sinusoidal like oscillations. While the solution from the Spalart-Allmaras model had a dominant, primary harmonic, the solution from the k-ω model contained multiple frequencies with varying magnitudes. The convergence behavior of both flow algorithms was nearly the same.

The second configuration studied was a flow problem consisting of three, vertical, square cylinders spanning a duct. This case was modeled after an experiment in which mean velocity and turbulence intensity profiles were available. Also for this configuration, only the k-ω turbulence model was used in conjunction with the two flow algorithms. The solution from the preconditioned Roe algorithm correlated slightly better with the available data than the artificial compressibility method. Like the single cylinder computations, both flow algorithms had very similar convergence behavior when run using the dual-time stepping procedure.

In summary, the predictions are more sensitive to the turbulence model choice than to the choice of algorithm. Using the Wilcox k-ω model, the preconditioned Roe algorithm produced more consistent results with experiment. The Spalart-Allmaras model overall produced better results with both algorithms than the Wilcox k-ω model.

The CFD simulations performed here were done using two-dimensional simulations. Future research could be done involving three-dimensional computations, which should do a better job at representing the physics since the flow is more three-dimensional due to turbulent structures. Additional turbulence models such as Reynolds Stress models should also be investigated.

Acknowledgments

The authors would like to thank the financial support from AEDC and the staff at AeroSoft for their technical support.
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